（問題６ １）

係数が実数の範囲でx^8+1を因数分解せよ。

（解答）

$$x^8+2x^4+1-2x^4$$

$$=(x^4+1)^2-2x^4$$

$$=\left(x^4+\sqrt{2}x^2+1\right)\left(x^4-\sqrt{2}x^2+1\right)$$

$$=\left\{(x^2+1)^2-\left(2-\sqrt{2}\right)x^2\right\}\left\{(x^2+1)^2-\left(2+\sqrt{2}\right)x^2\right\}$$

$$=\left(\boldsymbol{x^2+\sqrt{2-\sqrt{2}}x+1}\right)\left(\boldsymbol{x^2-\sqrt{2-\sqrt{2}}x+1}\right)\left(\boldsymbol{x^2+\sqrt{2+\sqrt{2}}x+1}\right)\left(\boldsymbol{x^2-\sqrt{2+\sqrt{2}}x+1}\right)$$