（問題８５）

$$\lim_{n\to\infty}\frac{(1+2+3+\cdots n)^5}{(1+2^4+3^4+\cdots n^4)^2}\text{を求めよ。}$$

（解答）

$$\frac{(1+2+3+\cdots n)^5}{(1+2^4+3^4+\cdots n^4)^2}$$

$$=\frac{\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\cdots+\frac{n}{n}\right)^5\times n^5}{\left(\left(\frac{1}{n}\right)^4+\left(\frac{2}{n}\right)^4+\left(\frac{3}{n}\right)^4+\cdots+\left(\frac{n}{n}\right)^4\right)^2\times n^8}$$

$$=\frac{\left\{\sum_{k=1}^{n}\frac{k}{n}\right\}^5}{n^3\times\left\{\sum_{k=1}^{n}\left(\frac{k}{n}\right)^4\right\}^2}$$

分母子を$n^5$で割ると

$$=\frac{\left\{\frac{1}{n}\sum_{k=1}^{n}\frac{k}{n}\right\}^5}{\left\{\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^4\right\}^2}$$

$$\lim_{n\to\infty}\frac{\left\{\frac{1}{n}\sum_{k=1}^{n}\frac{k}{n}\right\}^5}{\left\{\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^4\right\}^2}$$

$$=\frac{\left\{\int_0^1 xdx\right\}^5}{\left\{\int_0^1 x^4dx\right\}^2}$$

$$=\mathbf{\frac{25}{32}}$$