

偏微分方程式

$$\textcircled{1} \quad \frac{\partial u(x,y)}{\partial x} = 0$$

x に関する偏微分が 0。
は x によらない y だけの関数。
(答え) $u(x,y) = \phi(y)$

$$\textcircled{2} \quad \frac{\partial^2 u(x,y)}{\partial x \partial y} = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right) = 0$$

$$\frac{\partial u}{\partial y} = \phi_a(y)$$

$$\frac{\partial u}{\partial y} - \phi_a(y) = 0$$

$$\frac{\partial u}{\partial y} - \frac{\partial}{\partial y} \left\{ \int \phi_a(y) dy \right\} = 0$$

$$\frac{\partial}{\partial y} \left\{ u - \int \phi_a(y) dy \right\} = 0$$

$$u = \varphi(x) + \int \phi_a(y) dy$$

$$= \varphi(x) + \phi(y)$$

$$\textcircled{3} \quad \frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$$

$$\begin{cases} \alpha = x + y \\ \beta = x - y \end{cases}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \alpha} \frac{\partial \alpha}{\partial x} + \frac{\partial u}{\partial \beta} \frac{\partial \beta}{\partial x} = \frac{\partial u}{\partial \alpha} + \frac{\partial u}{\partial \beta} \left(\frac{\partial \alpha}{\partial x} = 1, \frac{\partial \beta}{\partial x} = 1 \right)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \alpha} \frac{\partial \alpha}{\partial y} + \frac{\partial u}{\partial \beta} \frac{\partial \beta}{\partial y} = \frac{\partial u}{\partial \alpha} - \frac{\partial u}{\partial \beta}$$

$$\therefore \frac{\partial u(\alpha, \beta)}{\partial \beta} = 0$$

$$u = \emptyset(\alpha)$$

$$= \boxed{\emptyset(x+y)}$$

$$\textcircled{4} \quad \frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right) = 0$$

$$\frac{\partial u}{\partial x} = \emptyset(y)$$

$$\frac{\partial u}{\partial x} \left\{ -\frac{\partial}{\partial x} \int \emptyset(y) dx \right\} = 0$$

$$\frac{\partial}{\partial x} \left\{ u - \int \emptyset(y) dx \right\} = 0$$

$$u - u - \int \emptyset(y) dx = \varphi(y)$$

$$\begin{aligned} u &= \varphi(y) + \int \emptyset(y) dx \\ &= \varphi(y) + x\emptyset(y) \end{aligned}$$

$$\textcircled{5} \quad 1 \text{ 次元波動方程式 } \frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}$$

変数変換

$$\begin{cases} \alpha = x - ct \\ \beta = x + ct \end{cases}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \alpha} \frac{\partial \alpha}{\partial x} + \frac{\partial u}{\partial \beta} \frac{\partial \beta}{\partial x}$$

$$= \frac{\partial u}{\partial \alpha} + \frac{\partial u}{\partial \beta}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial \alpha} \left(\frac{\partial u}{\partial \alpha} + \frac{\partial u}{\partial \beta} \right) \frac{\partial \alpha}{\partial x} + \frac{\partial}{\partial \beta} \left(\frac{\partial u}{\partial \alpha} + \frac{\partial u}{\partial \beta} \right) \frac{\partial \beta}{\partial x}$$

$$= \frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial \alpha \partial \beta} + \frac{\partial^2 u}{\partial \beta^2}$$

$$\frac{\partial u}{\partial t} = \frac{\partial \alpha}{\partial t} + \frac{\partial u}{\partial \beta} \frac{\partial \beta}{\partial t} = -c \left(\frac{\partial u}{\partial \alpha} - \frac{\partial u}{\partial \beta} \right)$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial \alpha} \left\{ -c \left(\frac{\partial u}{\partial \alpha} - \frac{\partial u}{\partial \beta} \right) \right\} \frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial \beta} \left\{ -c \left(\frac{\partial u}{\partial \alpha} - \frac{\partial u}{\partial \beta} \right) \right\} \frac{\partial \beta}{\partial t}$$

$$= c^2 \left(\frac{\partial^2 u}{\partial \alpha^2} - 2 \frac{\partial^2 u}{\partial \alpha \partial \beta} + \frac{\partial^2 u}{\partial \beta^2} \right)$$

$$\frac{\partial^2 u}{\partial \alpha \partial \beta} = 0$$

②と同じ

$$\begin{aligned} u(\alpha, \beta) &= \varphi(\alpha) + \emptyset(\beta) \\ &= \varphi(x - ct) + \emptyset(x + ct) \end{aligned}$$