

(問題 15)

(1)  $\cos \theta + \cos^2 \theta = 1$  のとき,  $\cos \theta$ 、 $\sin^2 \theta + 2\sin^4 \theta$  を求めよ。

(2)  $\sin \theta + \cos \theta = \frac{1}{\sqrt{5}}$  のとき,  $\tan^3 \theta + \frac{1}{\tan^3 \theta}$  を求めよ。

(解答)

(1)

$$\cos \theta = 1 - \cos^2 \theta \geq 0$$

$$\cos^2 \theta + \cos \theta - 1 = 0$$

$$\cos \theta = \frac{-1 \pm \sqrt{5}}{2}$$

$$\cos \theta = \frac{-1 + \sqrt{5}}{2} \quad (\cos \theta \geq 0 \text{ より})$$

$$\sin^2 \theta = 1 - \cos^2 \theta = \cos \theta = \frac{-1 + \sqrt{5}}{2}$$

$$\sin^2 \theta + 2\sin^4 \theta = \sin^2 \theta (1 + 2\sin^2 \theta)$$

$$= \frac{-1 + \sqrt{5}}{2} \left( 1 + 2 \cdot \frac{-1 + \sqrt{5}}{2} \right)$$

$$= \frac{5 - \sqrt{5}}{2}$$

(2)

$$\sin \theta + \cos \theta = \frac{1}{\sqrt{5}}$$

$$(\sin \theta + \cos \theta)^2$$

$$= 1 + 2 \sin \theta \cos \theta = \frac{1}{5}$$

$$\sin \theta \cos \theta = -\frac{2}{5}$$

$$\tan \theta + \frac{1}{\tan \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \frac{1}{\cos \theta \sin \theta} = -\frac{5}{2}$$

$$\tan^3 \theta + \frac{1}{\tan^3 \theta} = \left( \tan \theta + \frac{1}{\tan \theta} \right)^3 - 3 \left( \tan \theta + \frac{1}{\tan \theta} \right)$$

$$= -\frac{125}{8} - 3 \cdot \left(-\frac{5}{2}\right) = -\frac{65}{8}$$