

(問題 8)

2 次方程式 $x^2 - x + 1 = 0$ の解の一つを α とするとき $\alpha^{5800} + \alpha^{3500} + \alpha^{1700} + \alpha^{70}$ の値を求めよ。

(解答)

$$\alpha^2 - \alpha + 1 = 0 \text{ より } \alpha^3 = \alpha \cdot \alpha^2 = \alpha(\alpha - 1) = \alpha^2 - \alpha = -1$$

$$\alpha^{5800} = (\alpha^3)^{1933} \alpha = -\alpha$$

$$\alpha^{3500} = (\alpha^3)^{1166} \alpha^2 = \alpha^2$$

$$\alpha^{1700} = (\alpha^3)^{566} \alpha^2 = \alpha^2$$

$$\alpha^{70} = (\alpha^3)^{23} \alpha = -\alpha$$

$$\alpha^{5800} + \alpha^{3500} + \alpha^{1700} + \alpha^{70} = -\alpha + \alpha^2 + \alpha^2 - \alpha = -2$$